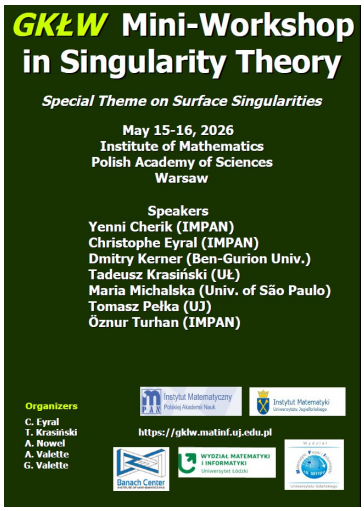
Gdańsk-Kraków-Łódź-Warszawa Mini-Workshop in Singularity Theory

Special Theme on Surface Singularities



15 – 16 May 2025

Room 6, ground floor
Institute of Mathematics of the Polish Academy of Sciences
ul. Śniadeckich 8, 00-656 Warsaw

Friday 15 May 2026

14:45 – 15:30 **Dmitry Kerner** (online) – *Separating sets on links of NSS*

Take the link of a normal surface singularity Link_r[X]. In the limit r→0 this Link_r[X] shrinks linearly except at a locus of real codimension one. This "separating set" vanishes at higher rate and captures the inner-Lipschitz geometry of X. I will explain how to detect these separating sets [and their topology] via polar curves, and how they determine various inner-Lipschitz invariants. Joint work with R. Mendes Pereira

15:30 – 16:00 **Coffe-break**

16:00 – 16:45 **Tadeusz Krasiński** (with cooperation of Sz. Brzostowski) – *The Łojasiewicz exponent of non-degenerate isolated singularities*

In the lecture I describe the problem of computing the Łojasiewicz exponent of non-degenerate isolated singularities f in terms of the Newton diagram of f. In particular, I present the recent results by B. Sigurdsson and F. Selyanin.

17:00 – 17:45 **Christophe Eyral** – *Introduction to Zariski pairs of surface singularities*

A conjecture of S. Yau asserts that if two isolated surface singularities in $\mathbb{C}^3$ have the same monodromy zeta-function and the same abstract topology, then they must also share the same embedded topology. This conjecture was disproved by E. Artal Bartolo, who explicitly constructed a pair made of two superisolated surface singularities with identical zeta-functions and abstract topologies but distinct embedded topologies. Today, any pair exhibiting this phenomenon is referred to as a *Zariski pair of surface singularities*. Subsequent research has further explored Zariski pair-like phenomena, resulting in several recent developments in the field. In this talk, and in the following one by Ö. Turhan, we will present a concise introduction to this subject and its main results.

18:00 – 18:45 **Öznur Turhan** – *New μ-Zariski pairs of surface singularities*

This talk is a continuation of the previous one by C. Eyral. To the best of our knowledge, all previously known examples of μ-, μ*-, link-, or ordinary Zariski pairs of surface singularities in $\mathbb{C}^3$ consist of (possibly weighted) Lê–Yomdin singularities. In this talk, we will present an example of a μ-Zariski pair involving surface singularities that are not of Lê–Yomdin type. This is a joint work with C. Eyral, M. Ishikawa and M. Oka.

Saturday 16 May 2026

9:30 – 10:15 **Maria Michalska** – *Lipschitz embeddings of algebraic curves*

We will show that a real algebraic curve has a semialgebraic Lipschitz normal embedding into the real plane if and only if as a graph it does not contain a subdivision of a Kuratowski graph. Moreover, there always exists a Lipschitz normal embedding as a real algebraic curve in $\mathbb{R}^3$. In the complex case we show that while semialgebraic Lipschitz normal embeddings exist, there may not exist Lipschtz normal embeddings as complex algebraic curves. This is joint work with Gustavo Menani and Zbigniew Jelonek, supported by FAPESP 2024/04171-9.

10:15 – 10:45 **Coffee-break**

10:45 – 11:30 **Tomasz Pełka** – *del Pezzo surfaces of rank one*

I will present an approach to classify log canonical del Pezzo surfaces of rank one. It is based on studying $\mathbb{P}^1$-fibrations of the minimal resolution which minimize the height, i.e. the intersection number of the fiber with the exceptional divisor. We prove that the height is at most 4 (with minor exceptions in characteristic 2 and 3) and the geometry becomes more rigid as the height grows. To illustrate this I will show a natural construction of some equisingular families of pairwise non-isomorphic del Pezzo surfaces of rank one and height 1 or 2, and explain why such families do not occur if the height is at least 3 (again, with minor exceptions). This is a joint work with Karol Palka.

11:45 – 12:30 **Yenni Cherik** – *Lipschitz Geometry of complex surface germs with non isolated singularities*

A germ of a complex analytic set $(X,0)$ is a subset of $\mathbb{C}^n$ defined as the common zero locus of a finite number of germs of holomorphic functions at the origin of $\mathbb{C}^n$. In this talk, we address the problem of the classification of such objects.

The problem of analytic classification (up to biholomorphism) of these sets appears to be out of reach. The topological classification (up to homeomorphism), on the other hand, is well understood thanks to the cone structure theorem, which makes it possible to construct a germ of homeomorphism between $(X,0)$ and the cone over the intersection of X with a real hypersphere of sufficiently small radius.

In this talk, we focus on Lipschitz classification, which lies between analytic and topological classification. The Lipschitz classification of germs of curves was completed through the work of Pham and Teissier in 1969, and later by Neumann and Pichon in 2013. The Lipschitz classification for the inner metric (distance defined by arc length) of surfaces with isolated singularities was completed by Birbrair, Neumann, and Pichon in 2014. Building on their theorem, as well as on earlier work, we present a Lipschitz classification for the inner metric of germs of complex surfaces with non-isolated surface singularities.