Gdańsk-Kraków-Łódź-Warszawa mini-workshop in Singularity Theory

30 - 31 May 2025

Room 403, 4th floor Institute of Mathematics of the Polish Academy of Sciences ul. Śniadeckich 8, 00-656 Warsaw



GKŁW mini-workshop

Friday 30 May 2025

11:00 - 11:45 Lev Birbrair

Lipschitz geometry of germs of real surfaces

Abstract – I am going to describe old and new results related to inner, outer and ambient Lipschitz geometry of germs of real semi-algebraic and definable surfaces. The subject is closely related to non-archimedean geometry and knot theory. I am going to make an overview. No preliminary knowledge is required.

11:50 – 12:35 Maria Michalska

Remarks on growth rates of polynomials restricted to subsets

Abstract – Fix a semialgebraic set S and a real function $g: S \rightarrow \mathbf{R}$. We address the question on how to determine for a polynomial *f* the power *m* such that the growth of *f* is bounded by g^m on the set S, up to a constant. We will review known results and focus on their constructive and explicit versions as well as show how Łojasiewicz exponent and resolution of singularities come into play. In particular, we will constructively describe degree of polynomials relative to any semialgebraic subset of the real plane. Moreover, we give the monomial generators for the algebra of bounded polynomials on basic semialgebraic set described by quasi-homogeneous inequalities. We will discuss how these results allow for certain sums of squares representations.

14:00 - 14:45 Stanisław Janeczko

Residual algebraic restrictions of differential forms

Abstract – We study germs of differential forms over singular varieties. The geometric restrictions of differential forms to singular varieties is introduced and algebraic restrictions of differential forms with vanishing geometric restrictions, called residual algebraic restrictions, are investigated. Residues of plane curves-germs, hypersurfaces, Lagrangian varieties as well as the geometric and algebraic restriction via a mapping were examined. This is a joint work with Goo Ishikawa.

14:50 – 15:35 Andrzej Weber

Characteristic classes of singular varieties: local and global

Abstract – This is a basic overview lecture on equivariant characteristic classes of singular varieties, with an emphasis on foundational ideas and localization techniques. I will demonstrate how localization tools for torus action apply to invariants such as the equivariant Chern-Schwartz-MacPherson class and provide an example of explicit computation. The goal is to illustrate how local data can be used to compute global geometric and topological invariants, and how global properties can, in turn, influence local behavior.

15:35 – 16:00 **Coffee break**

16:00 – 16:45 **Dmitry Kerner**

Equivalence of germs (of sets/mappings/foliations) over **k** vs that over **K**

Abstract – Consider real-analytic map-germs, $(\mathbf{R}^n, 0) \rightarrow (\mathbf{R}^m, 0)$. Two such germs can be complex-analytic equivalent, but not real-analytic equivalent. However, if the complex-analytic equivalence goes by coordinate changes whose linear parts are identities, then it implies the real-analytic equivalence. On the other hand, starting from complex map-germs, $(\mathbf{C}^n, 0) \rightarrow (\mathbf{C}^m, 0)$, and taking any further field extension $\mathbf{C} \subset \mathbf{K}$, the equivalence over \mathbf{K} implies that over \mathbf{C} .

I will give a more general (and stronger) version of this statement:

- for maps $X \rightarrow Y$ of (formal/analytic/algebraic) germs, with arbitrary singularities;
- for arbitrary extensions of fields, $\mathbf{k} \subset \mathbf{K}$. (In fact for faithfully-flat ring extensions);
- for a (right/left/left-right/contact) equivalence by unipotent elements.

As an application we get: if a *k*-analytic family is trivial over *K*, then it is trivial over *k*.

16:50 – 17:35 Jabub Koncki

SSM Thom polynomials of multisingularities

Abstract – Thom polynomial is a tool used for understanding the geometry of singular loci of maps. To a singularity germ η we associate a polynomial in infinitely many variables. Upon substituting these variables with the Chern classes of the relative tangent bundle of a stable map, we obtain the fundamental class of the η -singular loci of the given map. Thom polynomials have several generalizations, including extension to multisingularities, and polynomials that compute other cohomological properties of the singular loci, such as the Segre-Schwartz-MacPherson class.

In the talk, I will review these concepts and introduce the SSM-Thom polynomials of multisingularities. I will present a structure theorem for them that generalizes results of Kazarian.

The talk is based on a joint project with R. Rimányi.

9:00 – 9:45 **Zbigniew Jelonek**

Metric Zariski multiplicity conjecture for multiplicity two

Abstract – We show that if two algebraic (*n*-1)-dimensional cones $P, R \subset \mathbb{C}^n$ with isolated singularities are homeomorphic, then they have the same degree. We also prove that if two algebraic (*n*-1)-dimensional cones $P, R \subset \mathbb{C}^n$ are ambient homeomorphic, then their bases B_P and B_R have the same Euler characteristic. As an application we show that two bi-Lipschitz equivalent Brieskorn-Pham hypersurfaces have the same multiplicities at 0. As the second application we show our main result: if $(X,0), (Y,0) \subset (\mathbb{C}^n,0)$ are germs of analytic hypersurfaces, which are ambient bi-Lipschitz equivalent and $m_0(X)=2$, then also $m_0(Y)=2$. At the end of the talk we give also some application of our results to the Arnold-Vassiliew Problem.

09:50 – 10:35 **Öznur Turhan**

Newton-nondegenerate line singularities, Lê numbers and Bekka (c)-regularity

Abstract – Consider an analytic function f(t,z) defined in a neighbourhood of the origin of $\mathbf{C} \times \mathbf{C}^n$ such that for all t, the function $f_t(z):=f(t,z)$ defines a hypersurface of \mathbf{C}^n with a line singularity at $0 \in \mathbf{C}^n$. Denote by V(f) the hypersurface of $\mathbf{C} \times \mathbf{C}^n$ defined by f(t,z) and write Σf for its singular locus. We assume that f_t is "quasi-convenient" and Newton non-degenerate. Within this framework, we show that if the Lê numbers of f_t are independent of t for all small t, then Σf is smooth and $V(f) \setminus \Sigma f$ is Bekka (c)-regular over Σf . This is a version for line singularities of a result of Abderrahmane concerning isolated singularities.

As a corollary, we obtain that any family of quasi-convenient, Newton non-degenerate, line singularities with constant Lê numbers as above is topologically equisingular. In particular, this applies to families with non-constant Newton diagrams, and therefore extends, in some direction, a result previously observed by Damon.

This is a joint work with Christophe Eyral.

10:35 – 11:00 **Coffee break**

11:00 – 11:45 **Piotr Mormul**

From Kumpera's truncated multi-flags to moduli of the local classification of special 2-flags

Abstract – Antonio Kumpera is the father of the modern approach to classical Goursat flags (or: 1-flags, rank-2 geometric distributions together with their derived flags growing in ranks extremely modestly - always only by 1). He discovered (1978) singularities hidden in that seemingly regular world of geometrical structures. Around the turn of the century, when the local analysis of the Goursat flags was already well-advanced, Kumpera started to investigate multi-flags, departing from the analysis of certain underdetermined systems of PDEs. There soon emerged basic stratifications of the so-called special multi-flags, along with local pseudo-normal forms for distributions generating such flags. The status of

many real parameters entering those forms remained unclear, save for one example (2010) of a continuous modulus in flag's length 7. Very recently, with a substantial (mainly software) help from Andrzej Weber, continuous moduli of the local classification of special 2-flags have been found as early as in length 5. (The local classification in length 4 is known since 2010 to consist of 34 possible different local geometrical behaviours of those objects.)

11:50 – 12:35 Jean-Paul Brasselet

Characteristic classes under the Hirzebruch viewpoint: manifolds and singular varieties

Abstract – In a first part, I will give a survey on the Hirzebruch point of view to unify different characteristic classes of manifolds: the Chern class, the Todd class and the Thom–Hirzebruch's *L*-class. In a second part, I will show how to provide such unification of the three singular versions of these classes. That is a joint result with Shoji Yokura and Jörg Schürmann. In our paper we gave a conjecture that has been recently proved by J. Fernández de Bobadilla, I. Pallarés and M. Saito.