Gdańsk-Kraków-Łódź-Warszawa mini-workshop in Singularity Theory

3 – 4 October, 2025

Room 6, groundfloor Institute of Mathematics of the Polish Academy of Sciences ul. **Ś**niadeckich 8, 00-656 Warsaw



Friday 3 October 2025

14:30 – 15:30 Mutsuo Oka

Almost Newton non-degenerate function and some applications

Abstract — Let $f(\mathbf{z})$ be a Newton non-degenerate function at the origin. Then the zeta function of f is described combinatorially by Varchenko. We generalize this formula for "almost Newton non-degenerate function" where functions can have some Newton degenerate maximal faces. As an application, we consider a generalization of the Orlik–Milnor theorem. Let $f(z_1, ..., z_n)$ be a weighted homogeneous polynomial of degree d under a weight vector $P = {}^t(p_1, ..., p_n)$. If f has an isolated singularity at the origin, Orlik–Milnor gave a formula for the Milnor number $\mathbf{\mu}$. We consider a polynomial $g(\mathbf{z}) = f(\mathbf{z}) + h(\mathbf{z})$ such that f is a weighted homogeneous polynomial of degree d with one-dimensional singularity at the origin, and h is a polynomial with $\deg_p h = d + m$, $m \ge 1$, such that $S(f) \cap \{h_p\} = \{0\}$, so that g has an isolated singularity at the origin. For homogeneous polynomials with m = 1, Luengo, and for $m \ge 2$, Lê and Yomdin generalized the formula of Luengo. For generic weighted homogeneous polynomials with n = 3, Artal Bartolo, Luengo, Bodadilla, and Melle gave a formula for the Milnor number. In this talk, we generalize this formula for higher dimensions. I start from A'Campo's formula for the zeta function, Varchenko's formula, and then explain our formula with Christophe Eyral as an application of almost Newton non-degenerate functions.

15:30 – 16:00 **Coffee-break**

16:00 – 17:00 **Masaharu Ishikawa**

Milnor fibrations of type $f \overline{g}$ singularities and contact structures

Abstract – This is a survey of my results on Milnor fibrations of singularities of type $f \overline{g}$ and contact structures on the 3-sphere. Contact structures on the 3-sphere are classified into two types, called "tight" and "overtwisted". Milnor fibrations of complex polynomial functions support the tight contact structure on the 3-sphere. To study the contact structures

supported by Milnor fibrations of singularities of type $f \ \overline{g}$, explicit descriptions of the fiber surfaces and the contact structures are useful. As a consequence, we show that the supporting contact structures are always overtwisted. In the talk, I will explain some basic facts on contact structures and the ideas of the proof.

17:00 – 18:00 **Juliusz Banecki**

Uniform and uniform retract rationality

Abstract – In 1989, Gromov asked whether every smooth complex rational variety X is uniformly rational, meaning that each point of X admits a neighbourhood isomorphic to an open subset of the affine space. In the talk I will describe two results which solve the problem partially, and present their consequence in Oka theory.

18:00 – 19:00 **Yenni Cherik**

Lipschitz geometry of germs of complex surfaces

Abstract – It has been known since the work of Tadeusz Mostowski in 1985 that the set of germs of complex surfaces up to bilipschitz equivalence is countable. Building on the work of Lev Birbrair, Walter Neumann and Anne Pichon on the bilipschitz classification of complex surface germs, we will describe how to explicitly construct an infinite number of germs of complex surfaces with isolated singularity that are pairwise homeomorphic (and in fact with same normalization up to isomorphism) but that are not pairwise bilipschitz equivalent.

Saturday 4 October 2025

9:30 – 10:30 **Wojciech Domitrz**

Singularities of 2-dimensional indefinite improper affine spheres determined by singular curves

Abstract – This is a joint work with Michał Zwierzyński.

Improper affine spheres (IAS) are hypersurfaces whose affine Blaschke normal vectors are all parallel. They are given as the graphs of solutions of the classical Monge-Ampere equation. In 2013, F. Milan proved that 2-dimensional indefinite improper affine spheres are determined by their singular curves. We study singularities of IAS for generic singular curves.

10:30 – 11:30 Janusz Gwoździewicz

Polynomial submersions of degree 6

Abstract – We study polynomial submersions $p: \mathbb{R}^2 \to \mathbb{R}$ of degree 6 such that at least one fiber $p^{-1}(t)$ is not connected. Our aim is to show that every polynomial mapping $(p,q): \mathbb{R}^2 \to \mathbb{R}^2$

with nowhere vanishing Jacobian determinant such that deg $p \le 6$ is a global diffeomorphim.

11:30 – 11:45 **Pause**

11:45 – 12:45 **Dmitry Kerner**

Unfoldings of Maps(X,Y)

The Theory of Maps($(C^n,o),(C^m,o)$) was initiated by Whitney-Thom, and has flourished since then. One of its cornerstones is the theory of unfoldings of mapping-germs, with criteria of triviality and versality. I will report on the basics of the theory of Maps(X,Y), where the source&target are space-germs with arbitrary singularities. First come the relevant groupactions (right, contact, left-right), with their tangent spaces. Then I will give the criteria of triviality and versality of unfoldings.